NOVEMBER/DECEMBER 2024

23PPH33 — NUMERICAL METHODS AND COMPUTER PROGRAMMING - THEORY

Shanmuga Industries Arts & Science College T.V.Malai

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is the Newton-Raphson formula for finding roots?

2. Identify one advantage of the Newton-Raphson method over the Bisection method.

3. How to represent a system of linear equations using matrices?

4. Demonstrate the limitation of the matrix inversion method by providing an example where it fails.

5. Define the term "residual" in the context of curve fitting.

6. What is Newton's backward interpolation formula used for in interpolation?

7. Write Simpson's rule for approximating the integral of a function $f(x)$ over an interval $[a,b]$ with step size h.

8. Give formula for the 2-point Gauss-Legendre quadrature for numerically evaluating the integral $\int_{-1}^{1} f(x)dx$.

9. Use Newton-Raphson method for finding the zeros of the function $f(x) = e^x - 3x$.

10. Write the output of a program that uses the Trapezoidal rule to approximate the integral of $f(x) = x^2$ over [0, 1].

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SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Using the Bisection method, find an approximation to the root of $f(x) = x^3 - 4x + 1$ in the interval [1, 2] after two iterations.

Or

(b) Derive the Newton-Raphson method formula from the Taylor series expansion.

19. Solve the initial value problem $dy/dx = x + y$ with $y(0) = 1$ using the 4th order Runge-Kutta method with a step size $h = 0.2$ up to $x = 0.4$.

20. Formulate a program to solve an ordinary differential equation (ODE) using the fourth-order Runge-Kutta method.

12. (a) Explain the matrix inversion method to solve the following system of simultaneous equations. $2x + 3y = 5$, $4x - y = 1$.

Or

(b) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ using the Gauss – Jordan method.

13. (a) Given the following table of values :

X	Y
1	2
2	4
3	6
4	8
5	10

Use Newton's forward interpolation formula to estimate the value of y at $x = 2.5$.

Or

(b) Explain the method of least squares and use it to fit a linear polynomial to the data points (1, 2) (2, 4) and (3, 6).

14. (a) Given the function $f(x) = x^3 - 2x^2 + x$, use numerical differentiation to estimate $f'(2)$ using the central difference formula with a step size $h = 0.01$.

Or

(b) Give the advantages of Simpson's rule over the Trapezoidal rule for numerical integration. Use appropriate error estimates in your comparison.

15. (a) Write a program for implementing the Bisection method to find the zeros of a polynomial.

Or

(b) Use Newton's backward interpolation for the following set of data : $(x_1 = 1, y_1 = 2)$, $(x_2 = 3, y_2 = 3)$, $(x_3 = 3, y_3 = 5)$ using a program.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Using the Bisection method, find the root of $f(x) = x^3 - 4x + 1$ in the interval [1, 2]. Perform four iterations and show all the steps in detail.

17. Explain the matrix inversion method to solve a system of linear equations. Use it to solve the system : $2x + y - z = 8$, $-3x - y + 2z = -11$, $-2x + y + 2z = -3$

18. Given the following data points :

x	y
1	2
2	3
3	5
4	7
5	11

Use the method of least squares to fit a linear polynomial $y = ax + b$ to the data points. Find the values of a and b.